THE LEARNING TREE

PRACTICE PAPER – 1

MATRIX, DETERMINANT, INVERSE TRIGONOMETRY, CONTINUITY & DIFFERENTIABILITY, APPLICATION OF DERIVATIVE, PROBABILITY, LINEAR PROGRAMMING

Time Allowed: 3 hours   Maximum Marks: 80

General Instructions:

(i) All questions are compulsory.
(ii) This practice test paper contains 36 questions.
(iii) Questions 1 to 20 in Section – A are very short answer type questions carrying 1 mark each.
(iv) Questions 21 to 26 in Section – B are short answer type questions carrying 2 marks each.
(v) Questions 27 to 32 in Section – C are long answer - I type questions carrying 4 marks each.
(vi) Questions 33 to 36 in Section – D are long answer - II type questions carrying 6 marks each.

Section – A

Q.1 Principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is

(a) $\frac{\pi}{3}$ (b) - $\frac{\pi}{3}$ (c) $\frac{5\pi}{6}$ (d) $-\frac{\pi}{6}$

Q.2 If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}, then\ the\ value\ of\ x + y^2 + z^3$ is

(a) 1   (b) 3   (c) 2   (d) 5

Q.3 If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}\ and\ A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & c \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}\ then$

(a) a = 2, c = $\frac{1}{2}$ (b) a = 1, c = -1   (c) a = -1, c = 1   (d) a = $\frac{1}{2}$, c = $\frac{1}{2}$

Q.4 If $\begin{bmatrix} 5 & 0 \\ 0 & 7 \end{bmatrix}^{-1}\begin{bmatrix} x \\ -y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, then

(a) x = 5, y = 14   (b) x = -5, y = 14   (c) x = -5, y = -14   (d) x = 5, y = -14

Q.5 $If\ y = e^{\frac{1}{2}\log(1+tan^2x)}, then\ \frac{dy}{dx}\ is\ equal\ to$

(a) $\frac{1}{2}sec^2x$ (b) sec^2x (c) sec x tan x   (d) $e^{\frac{1}{2}\log(1+tan^2x)}$

Q.6 Differentiate e^x with respect to $\sqrt{x}$.

(a) -2$e^x\sqrt{x}$ (b) - $e^x\sqrt{x}$ (c) $e^x\sqrt{x}$ (d) 2$e^x\sqrt{x}$

Q.7 If $f(x) = x^3 - 6x^2 + x + 3$ be a decreasing function, then x lies in

(a) $(-\infty, -1) \cap (3, \infty)$ (b) $(1, 3)$ (c) $(3, \infty)$ (d) None of these

Q.8 The distance 's' meters covered by a body in t seconds is given by $s = 3t^2 - 8t + 5$. The body will stop after

(a) 1 second (b) ¾ second (c) 4/3 seconds (d) 4 seconds

Q.9 Let A and B be two given events such that P(A) = 0.6, P(B) = 0.2 and $P\left(\frac{A}{B}\right) = 0.5$. Then $P\left(\frac{A'}{B'}\right)$ is

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{3}{8}$ (d) $\frac{6}{7}$

Q.10 A bag contains 5 red, 6 blue and 4 black balls. Three balls are drawn from the bag. Then the probability that none of them is red, is

(a) $\frac{24}{91}$ (b) $\frac{2}{91}$ (c) $\frac{6}{35}$ (d) None of these

Q.11 Let A and B be two events. If P(A) = 0.2, P(B) = 0.4, P(A∪B) = 0.6, then find $P\left(\frac{A}{B}\right)$.

Q.12 Let A and B be two events. If P(A) = 0.4, P(B) = 0.3, P(A∪B) = 0.5, then find $P(B' \cap A)$.

Q.13 If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1} x\right) = 1, then\ find\ the\ value\ of\ x.$

Q.14 Solve: $\cot^{-1} x + \tan^{-1} 8 = \frac{\pi}{2}$

Q.15 If A is a square matrix of order 3 such that $|A| = 3$, then find $|A.adj.A|$

Q.16 If A and B are square matrices of order 3 such that $|A| = -1\ and\ |B| = 3$, then find the value of $|7AB|$.

Q.17 Prove that the function e^{2x} is strictly increasing on R.

Q.18 Find the derivative of $\sin(\sin x^2)\ at\ x = \sqrt{\frac{\pi}{2}}$.

Q.19 Discuss the continuity of the function f(x) = sin x – cos x.

Q.20 The total revenue received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when x = 7.

Section – B

Q.21 For what value of k, the following function is continuous at x =0?

$$F(x) = \begin{cases} \frac{1-cos2x}{2x^2}, x \neq 0 \\ k, \quad x = 0 \end{cases}$$

Q.22 Find the approximate value of f(2.01), where $f(x) = 4x^2 + 5x + 2$.

Q.23 A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event that the 'number is even' and B be the event that the 'number is red'. Are A and B independent?

Q.24 $If\ y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \cdots to\ \infty}}}, prove\ that\ \frac{dy}{dx} = \frac{\cos x}{2y-1}$

Q.25 Prove that : $tan^{-1}\left(\frac{m}{n}\right) - tan^{-1}\left(\frac{m-n}{m+n}\right) = \frac{\pi}{4}$

Q.26 Prove that all the diagonal elements of a skew symmetric matrix are equal to zero.

Section – C

Q.27 Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x, 0<x<1$

OR

Prove that : $\tan\left(\frac{\pi}{4}+\frac{1}{2}cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$

Q.28 Using properties of determinant, prove that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

OR

If A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, prove that A^n = $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, where n is a positive integer.

Q.29 Prove that $\frac{d}{dx}\left\{\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right\} = \sqrt{a^2-x^2}$

OR

If the function f(x) = $\begin{cases} 3ax+b, if\ x>1 \\ 11, if\ x=1 \\ 5ax-2b, if\ x<1 \end{cases}$ is continuous at x = 1, find the values of a and b.

Q.30 Water is dripping out from a conical funnel of semi-vertical angle $\frac{\pi}{4}$ at a uniform rate of 2 cm^2/sec in its surface area through a tiny hole at the vertex in the bottom. When the slant height of water is 4 cm, find the rate of decrease of the slant height of water.

OR

Find the equation of tangent to the curve $y=\frac{(x-7)}{(x-2)(x-3)}$ at the point where it meets x-axis.

Q.31 Find the intervals in which f(x) = sin x - cos x, where $0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

OR

It is given that for the function f given by f(x) = $x^3 + bx^2 + ax$, x ∈ $[1, 3]$

Rolle's theorem holds with c = $2+\frac{1}{\sqrt{3}}$. Find the values of a and b.

Q.32 A person has undertaken a construction job. The probabilities are 0.65 that their will be strike, 0.80 that the construction job will be completed in time if there is no strike, and 0.32 that the construction job will be completed in time if there is strike. Determine the probability that the construction job will be completed in time.

OR

A bag contains 4 balls. Two balls are drawn at random and are found to be white. What is the probability that all balls are white?

Section – D

Q.33 A wire of length 28m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces to that the combined areas of the square and the circle is minimum.

OR

Show that semi – vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

Q.34 A man spends on petrol of his motor cycle Rs. 2 per km, if he rides at 25 km/hr and Rs. 5 per km, if he rides at 40 km/hr. Find the maximum distance that he can travel in one hour if he wishes to spend Rs. 100. Formulate the L.P.P. and solve it graphically.

OR

A toy company manufactures two types of dolls. A and B market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximize the profit?

Q.35 Using elementary transformation find the inverse of the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

OR

Two schools P and Q want to award their selected students for values of sincerity, truthfulness and hard work at the rate of Rupees x, Rupees y and Rupees z for each respective value per student. School P awards its 2, 3 and 4 students on the above respective values with a total prize money of Rs. 4600. School Q wants to award is 3, 2 and 3 students on the respective values with a total award money of Rs. 4100. If the total amount of the award money for one prize on each value is Rs. 1500, using matrices find out the award money for each value.

Q.36 Suppose that the reliability of a HIV test is specified as follows: Of people having HIV, 90% of the tests detect the disease but 10% go undetected. Of people free of HIV, 99% of the tests are judged HIV –ve but 1% are diagnosed as showing HIV +ve. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV +ve. What is the probability that the person actually has HIV?

OR

In answering a question on a multiple choice test, a student either knows answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?